Equilíbrio geral: produção

1. Eficiência

1.1 Um consumidor e dois bens

1 Consumidor $\quad U(x_1, x_2) \quad\quad (w_1, w_2) =$ dot. inicial.

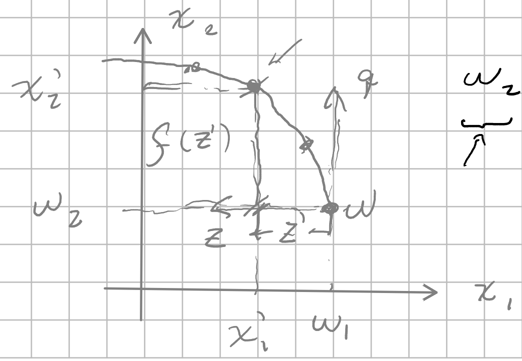
2 bens

1 função de produção p/ o bem 2 ; $q_2 = f(z)$

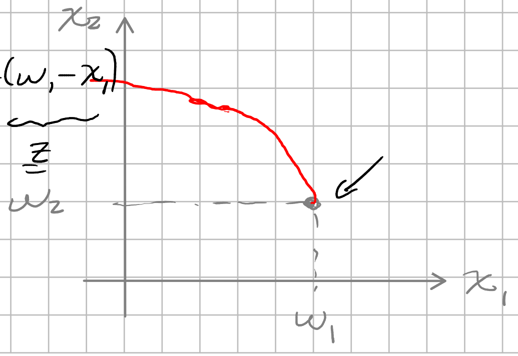
↳ qdade usada como insumo na prod. do bem 2.

$$x_1 \leq w_1 - z \Rightarrow z = w_1 - x_1$$

$$0 \leq x_1 \leq w_1 - z \quad\quad 0 \leq x_2 \leq w_2 + f(z) \quad ; \quad z \geq 0$$



$\underbrace{w_2}_{\uparrow} + f(\underbrace{w_1 - x_1}_{z})$



- Cond. de eficiência:

$$\text{Max } U(x_1, x_2)$$

suj. a

$$0 \le x_1 = \omega_1 - z_1 \le \omega_1 \Rightarrow z = \omega_1 - x_1$$

$$0 \le x_2 = \omega_2 + f(z) \Leftarrow \Rightarrow \underbrace{x_2 = \omega_2 + f(\omega_1 - x_1)}$$

$$\underset{x_1 \ge 0}{\text{Max}} \ \underbrace{U(x_1, \omega_2 + f(\omega_1 - x_1))}$$

cond. 1ª ordem

$$\underbrace{\frac{\partial U(x_1^*, x_2^*)}{\partial x_1}}_{UMg_1} - \underbrace{\frac{\partial U(x_1^*, x_2^*)}{\partial x_2}}_{UMg_2} \underbrace{\frac{d f(\omega_1 - x_1^*)}{dz}}_{PMg} = 0$$

$UMg_1 \, dx_1$

$UMg_2 \, PMg \, dx_1$

$$\left[\begin{array}{l} UMg_1 = UMg_2 \, PMg \\ \dfrac{UMg_1}{UMg_2} = PMg \end{array}\right.$$

caso $0 < x_1^* < \omega_1$

$UMg_1 \le UMg_2 \, PMg$

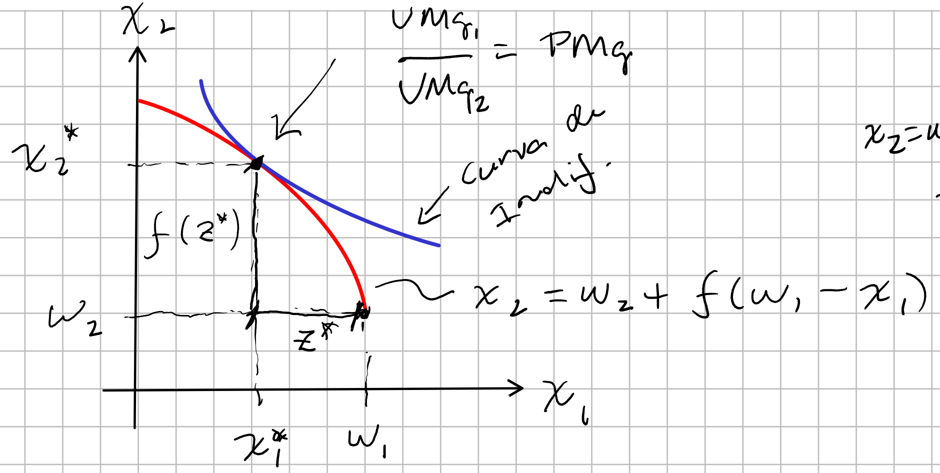
$\dfrac{UMg_1}{UMg_2} \le PMg$

caso $x_1^* = 0$

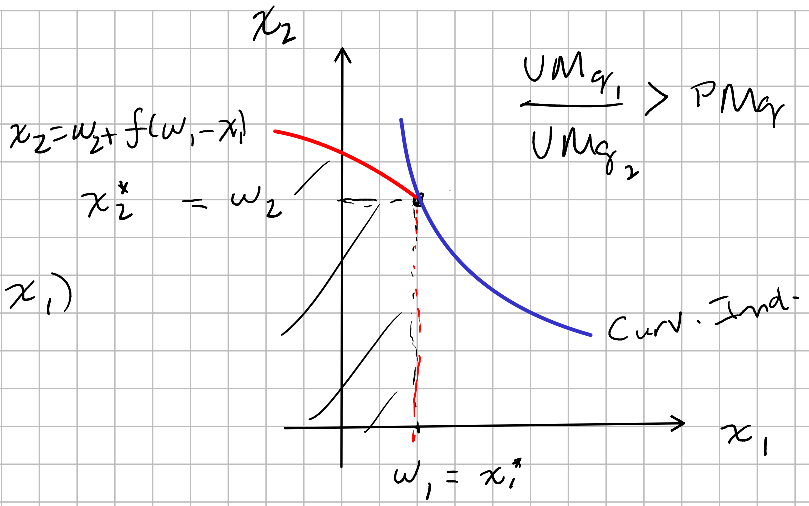
$UMg_1 \ge UMg_2 \, PMg$

$\dfrac{UMg_1}{UMg_2} \ge PMg$

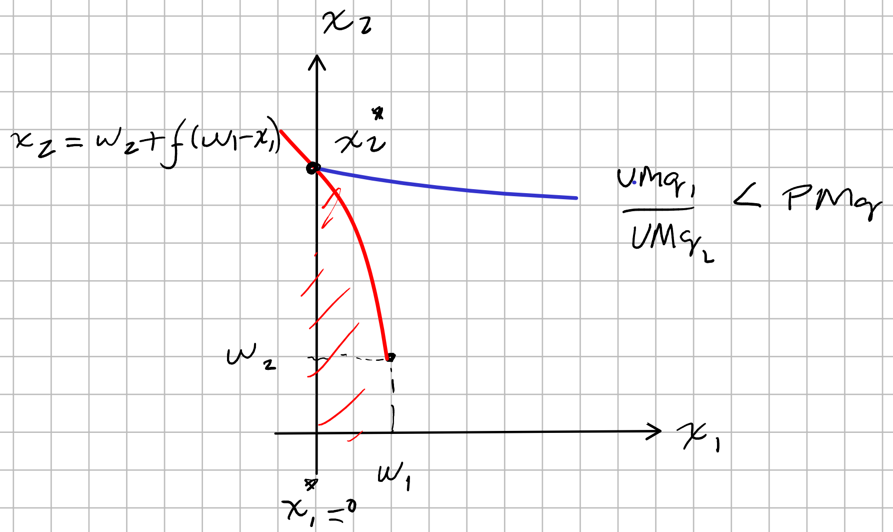
caso $x_1^* = \omega_1$



x_2
$\frac{UMg_1}{UMg_2} = PMg$
x_2^*
curva de Indif.
$f(z^*)$
w_2
$x_2 = w_2 + f(w_1 - x_1)$
z^*
x_1
x_1^*
w_1



x_2
$\frac{UMg_1}{UMg_2} > PMg$
$x_2 = w_2 + f(w_1 - x_1)$
$x_2^* = w_2$
Curv. Ind.
x_1
$w_1 = x_1^*$



x_2
$x_2 = w_2 + f(w_1 - x_1)$
x_2^*
$\frac{UMg_1}{UMg_2} < PMg$
w_2
x_1
w_1
$x_1^* = 0$

1.2 Um consumidor e ℓ bens.

dot. inicial $= (\omega_1, \omega_2, \ldots, \omega_\ell)$

Função de util: $U(x_1, x_2, \ldots, x_\ell) = U(x)$

Funções de produção

$q_1 = f_1(z_{1,1}, z_{1,2}, \ldots, z_{1,\ell})$

$z_{1,h}$ = qdade do bem h empregado na prod. do bem 1.

$q_2 = f_2(z_{2,1}, z_{2,2}, \ldots, z_{2,\ell})$

$\vdots$

$q_h = f_h(z_{h,1}, z_{h,2}, \ldots, z_{h,\ell})$

$z_{h,k}$ = qdade do bem k empregado na produção do bem h

$\vdots$

$q_\ell = f_\ell(z_{\ell,1}, z_{\ell,2}, \ldots, z_{\ell,\ell})$

$$0 \leq x_1 = \omega_1 - \sum_{h=1}^{\ell} z_{h,1} + f_1(z_{1,1}, z_{1,2}, \ldots, z_{1,\ell})$$

$$0 \leq x_h = \omega_h - \sum_{k=1}^{\ell} z_{k,h} + f_h(z_{h,1}, z_{h,2}, \ldots, z_{h,\ell})$$

$$z_{hk} \geq 0 \qquad h, k = 1, \ldots, \ell$$

$$\text{Max } U\Big(\omega_1 - \sum_{h=1}^{\ell} z_{h,1} + f_1(z_{1,1}, z_{1,2}, \dots, z_{1,\ell}),$$

$$\omega_2 - \sum_{h=1}^{\ell} z_{h,2} + f_2(z_{2,1}, z_{2,2}, \dots, z_{2,\ell}),$$

$$\vdots$$

$$\omega_k - \sum_{h=1}^{\ell} z_{hk} + f_k(z_{k,1}, z_{k,2}, \dots, z_{k,\ell}),$$

$$\vdots$$

$$\omega_\ell - \sum_{h=1}^{\ell} z_{h,\ell} + f_\ell(z_{\ell,1}, z_{\ell,2}, \dots, z_{\ell\ell})\Big)$$

$$\omega_h - \sum_{k=1}^{\ell} z_{h,k} + f_h(z_{h,1}, \dots, z_{h,k}, \dots, z_{h,\ell})$$

cond. 1ª ordem:

$$\Rightarrow \frac{dU}{dz_{hk}} = 0 \quad \text{caso} \quad z^*_{h,k} > 0 \quad p/\ h = 1, \dots, \ell \ \text{e} \ k = 1, \dots, \ell$$

$$(x^*_k, x^*_h > 0)$$

$$\frac{dU}{dz_{hk}} = -\frac{\partial U}{\partial x_k} + \frac{\partial U}{\partial x_h}\frac{\partial f_h}{\partial z_{hk}} = 0 \Rightarrow -UMg_k + UMg_h PMg_{hk} = 0$$

PMg_{hk}

prod. marginal do bem k na produção do bem h.

$$\underline{UMg_h\, PMg_{hk}} = \underline{UMg_k}$$

- UMg_k : Variação de utilidade por unidade do bem k consumido na margem
- $UMg_h\, PMg_{hk}$: Variação de utilidade por unidade do bem k empregado, na margem, na produção do bem h.

Se $UMg_k > UMg_h\, PMg_{hk}$ vale a pena reduzir o uso do bem k como insumo na producão do bem h até que $\underline{UMg_k = UMg_h\, PMg_{hk}}$ ou até que $\underline{z_{hk}} = 0$, o que ocorrer primeiro

Se $UMg_k < UMg_h\, PMg_{hk}$ vale a pena reduzir o consumo do bem k para aumentar seu emprego na produção do bem h até que $\underline{UMg_k = UMg_h\, PMg_{hk}}$ ou até que o consumo do bem k seja zerado, o que ocorrer primeiro.

$(x_k^* = 0)$

$$\underbrace{x_k = \omega_k - \sum_{h=1}^{l} z_{hk} + f_k(z_{k1}, \dots, z_{kl})}$$

∴ as condições de 1ª ordem são

$UMg_k = UMg_h\, PMg_{hk}$ ou $\dfrac{UMg_k}{UMg_h} = PMg_{hk}$ caso $x_k^* > 0$ e $z_{hk}^* > 0$,

$UMg_k \geq UMg_h\, PMg_{hk}$ ou $\dfrac{UMg_k}{UMg_h} \geq PMg_{hk}$ caso $z_{hk}^* = 0$ e $x_k^* > 0$

$UMg_k \leq UMg_h\, PMg_{hk}$ ou $\dfrac{UMg_k}{UMg_h} \leq PMg_{hk}$ caso $x_k^* = 0$ e $z_{hk}^* > 0$

+ 2 bens: q e t $\quad x_k^*, x_h^*, x_q^*, x_t^* > 0 \quad z_{hk}^*, z_{hq}^*, z_{tk}^*, z_{tq}^* > 0$

(i) $UMg_k = UMg_h\, PMg_{hk}$

(ii) $UMg_k = UMg_t\, PMg_{tk}$ $\Rightarrow 1 = \dfrac{UMg_h}{UMg_t}\,\dfrac{PMg_{hk}}{PMg_{tk}} \Rightarrow \boxed{\dfrac{UMg_t}{UMg_h}} = \boxed{\dfrac{PMg_{hk}}{PMg_{tk}}}$

$t = 1$

$h = 2$

$k = 3$

(iii) $UMg_q = UMg_h\, PMg_{hq}$

TMS_{th} = Taxa Marginal de Transformação

$\dfrac{(i)}{(iii)}: \dfrac{UMg_k}{UMg_q} = \dfrac{PMg_{hk}}{PMg_{hq}} \Rightarrow TMS_{kq} = TMST_{hkq}$

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$\dfrac{UMg_1}{UMg_2} = \dfrac{PMg_{2,3}}{PMg_1}$

$(i)\quad UMg_k = UMg_h \, PMg_{hk}$

$(ii)\quad UMg_k = UMg_t \, PMg_{tk}$

$(iii)\quad UMg_q = UMg_h \, PMg_{hq}$

$(iv)\quad UMg_q = UMg_t \, PMg_{tq}$

$\frac{(i)}{(iii)} : \frac{UMg_k}{UMg_q} = \frac{PMg_{hk}}{PMg_{hq}}$

$\frac{(ii)}{(iv)} , \frac{UMg_k}{UMg_q} = \frac{PMg_{tk}}{PMg_{tq}}$

$TMST_{hkq} = TMST_{tkq}$

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Ilustração gráfica ①

$$u(x_1, x_2, x_3) = v(x_1, x_2)$$

$\omega_1 = 0, \ \omega_2 = 0, \ \omega_3 > 0$

$\omega_1, \omega_2, \omega_3$

$$\frac{UMg_1}{UMg_2} : \frac{\cancel{\text{util.}} / \text{uni. bem 1}}{\cancel{\text{util.}} / \text{uni. bem 2}} = \frac{\text{uni. bem 2}}{\text{uni. bem 1}}$$

$q_1 = f_1(z_{1,3}) \ ; \quad q_2 = f_2(z_{2,3})$

$f_1'(z_{1,3}), \ f_2'(z_{2,3}) > 0$

$f_1''(z_{1,3}), \ f_2''(z_{2,3}) < 0$

restrições

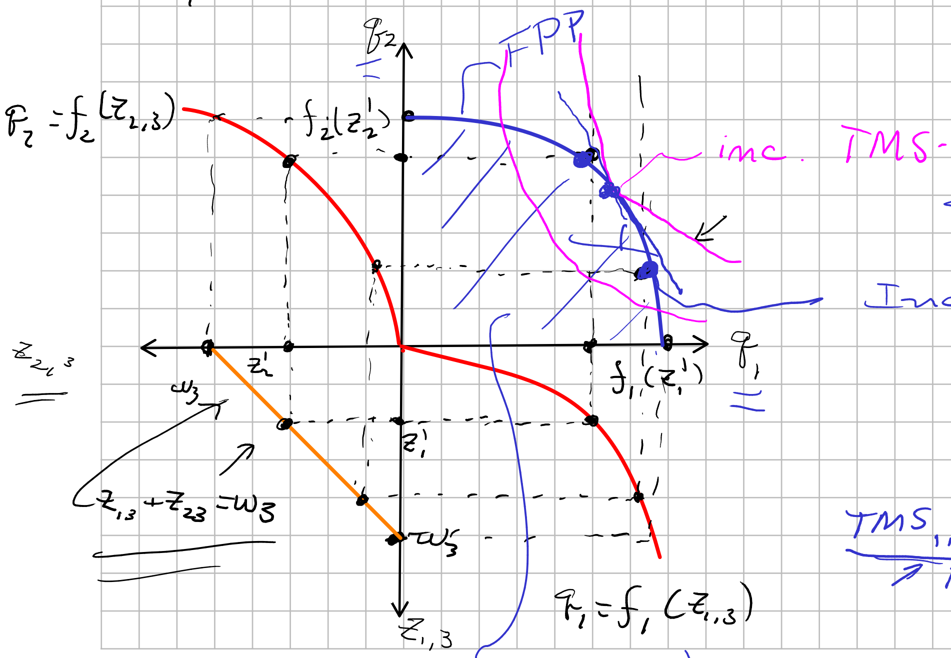
$z_{1,3}, \ z_{2,3} \geq 0$

$0 \leq x_1 = \omega_1 + f_1(z_{1,3})$

$0 \leq x_2 = \omega_2 + f_2(z_{2,3})$

$0 \leq x_3 = \omega_3 - z_{1,3} - z_{2,3}$

$\underbrace{z_{1,3} + z_{2,3} = \omega_3}$



"Fatia" do conj. de possibilidades de produção

Incl. de FPP = TMT

$$= \frac{PMg_{2,3}}{PMg_{1,3}} : \frac{\cancel{\text{un. bem 2}} / \cancel{\text{un. bem 3}}}{\text{un. bem 1} / \cancel{\text{un. bem 3}}} = \frac{\text{un. bem 2}}{\text{un. bem 1}}$$

$TMS_{1,2} = TMT_{1,2}$

$$\begin{cases} q_2 = f_2(\omega_3 - z_{1,3}) \\ q_1 = f_1(z_{1,3}) \end{cases}$$

$$\frac{dq_2}{dq_1} = -f_2'(\omega_3 - z_{1,3}) \frac{dz_{1,3}}{dq_1}$$

$$\frac{dq_2}{dq_1} = -\frac{f_2'(\cdot)}{f_1'(\cdot)} = -\frac{PMg_{2,3}}{PMg_{1,3}}$$

$$1 = f_1'(z_{1,3}) \frac{dz_{1,3}}{dq_1} \Rightarrow \frac{dz_{1,3}}{dq_1} = \frac{1}{f_1'(z_{1,3})}$$

Ilustração gráfica (II):

4 bens com dotações iniciais $\omega_1 = 0$, $\omega_2 = 0$, $\underline{\underline{\omega_3}} > 0$ e $\underline{\underline{\omega_4}} > 0$

Função de utilidade: $U(x_1, x_2)$

Funções de produção: $q_1 = f_1(z_{1,3}, z_{1,4})$ e $q_2 = f_2(z_{2,3}, z_{2,4})$

Produções tecnicamente eficientes:

$$\max_{z_{13}, z_{14}, z_{23}, z_{24}} f_1(z_{13}, z_{14}) \quad \text{suj. a} \quad z_{13}, z_{14}, z_{23}, z_{24} \geq 0 \quad \text{e} \quad f_2(z_{2,3}, z_{24}) \geq q_2$$

$$\underline{z_{1,3} + z_{2,3} = \omega_3} \quad \text{e} \quad \underline{z_{1,4} + z_{2,4} = \omega_4}$$

$$\mathcal{L} = f_1(z_{1,3}, z_{1,4}) + \lambda\left[f_2(z_{23}, z_{24}) - q_2\right] - \mu_3(z_{13} + \underline{z_{2,3}} - \omega_3) - \mu_4(z_{1,4} + z_{2,4} - \omega_3)$$

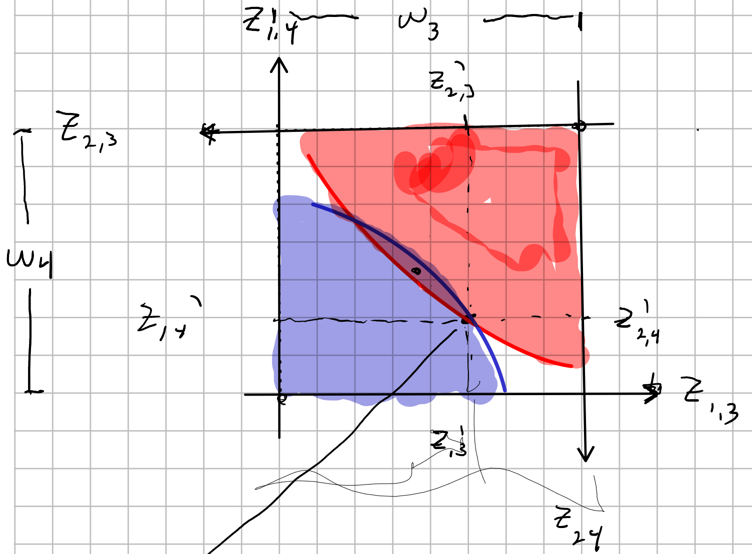
Cond. 1ª ordem: (além das restrições)

$$\frac{\partial \mathcal{L}}{\partial z_{13}} = 0 \Rightarrow \frac{\partial f_1}{\partial z_{1,3}} - \mu_3 = 0 \qquad \frac{\partial \mathcal{L}}{\partial z_{23}} = 0 \Rightarrow \lambda \frac{\partial f_2}{\partial z_{23}} - \mu_3 = 0 \qquad \lambda \frac{\partial f_2}{\partial z_{23}} = \frac{\partial f_1}{\partial z_{13}}$$

$$\frac{\partial \mathcal{L}}{\partial z_{14}} = 0 \Rightarrow \frac{\partial f_1}{\partial z_{14}} - \underline{\mu_4} = 0 \qquad \frac{\partial \mathcal{L}}{\partial z_{24}} = 0 \Rightarrow \lambda \frac{\partial f_2}{\partial z_{24}} - \underline{\mu_4} = 0 \qquad \lambda \frac{\partial f_2}{\partial z_{24}} = \frac{\partial f_1}{\partial z_{14}}$$

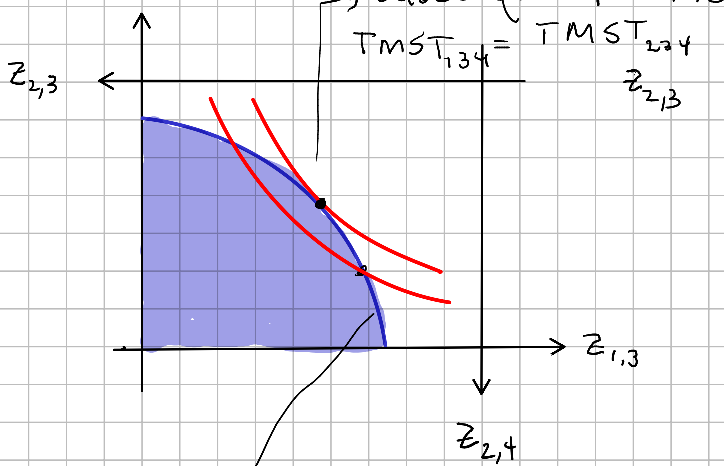
$$\left.\begin{aligned} \lambda \frac{\partial f_2}{\partial z_{23}} &= \frac{\partial f_1}{\partial z_{1,3}} \\ \lambda \frac{\partial f_2}{\partial z_{24}} &= \frac{\partial f_1}{\partial z_{1,4}} \end{aligned}\right\} \quad \frac{\partial f_2/\partial z_{23}}{\partial f_2/\partial z_{24}} = \frac{\partial f_1/\partial z_{13}}{\partial f_1/\partial z_{1,4}} \;;\quad \frac{PMg_{23}}{PMg_{24}} = \frac{PMg_{13}}{PMg_{14}} \;;\quad TMST_{2,3,4} = TMST_{1,3,4}$$

Caixa de Edgeworth na produção

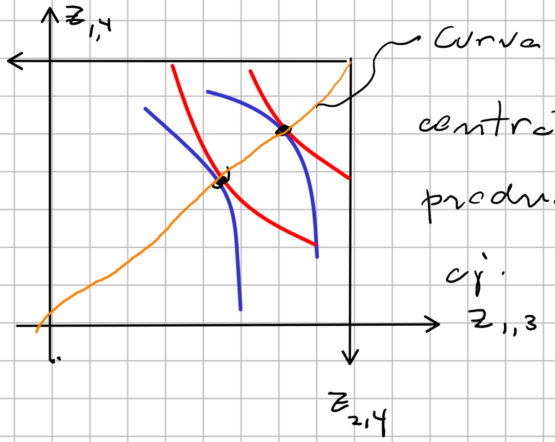


→ alocação técnicamente ineficiente.

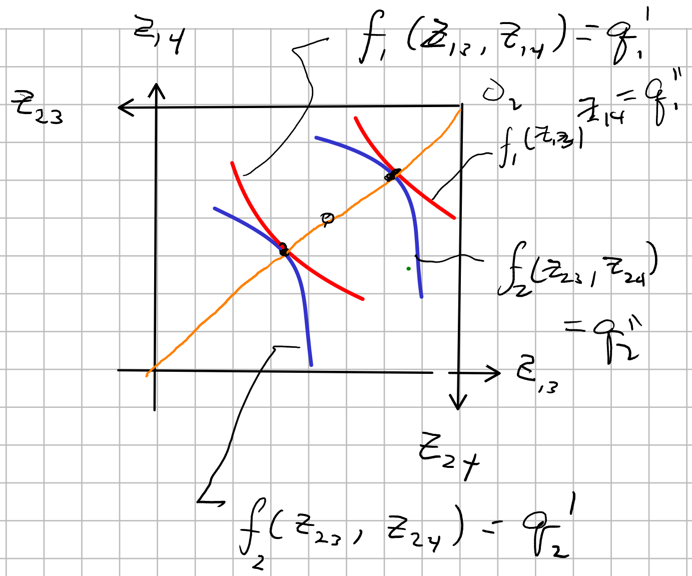
→ alocação tecnicamente eficiente

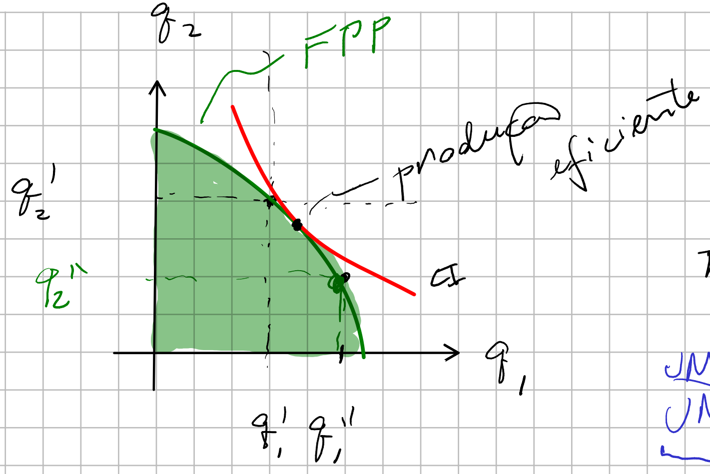


$f_2(z_{23}, z_{24}) = q_2$



Curva de contrato na produção ou cj. de Pareto na Produção.





Front. de possibilidades de produção

TMS = Inclin. da FPP

$$TMS_{1,2} = \frac{PMg_{23}}{PMg_{13}} = \frac{PMg_{24}}{PMg_{14}} = TMT_{1,2}$$

$$\frac{UMg_1}{UMg_2} = \frac{PMg_{23}}{PMg_{13}}$$

$$\frac{UMg_h}{UMg_q} = \frac{PMg_{fh}}{PMg_{fq}}$$

$\frac{dq_2}{dq_1}$ calculada sobre a FPP:

$$\frac{PMg_{13}}{PMg_{14}} = \frac{PMg_{23}}{PM_{2,4}}$$

$$\frac{PMg_{23}}{PMg_{13}} = \frac{PMg_{24}}{PMg_{14}} = -\lambda$$

$$w_3 = z_{1,3} + z_{2,3}$$

$$w_4 = z_{1,4} + z_{2,4}$$

$$q_1 = f_1(z_{1,3}, z_{1,4})$$

$$q_2 = f_2(z_{2,3}, z_{24})$$

$$\frac{dz_{13}}{dq_1} + \frac{dz_{23}}{dq_1} = 0$$

$$\frac{dz_{14}}{dq_1} + \frac{dz_{24}}{dq_1} = 0$$

$$PMg_{13}\frac{dz_{13}}{dq_1} + PMg_{14}\frac{dz_{14}}{dq_1} = 1$$

$$PMg_{23}\frac{dz_{23}}{dq_1} + PMg_{24}\frac{dz_{24}}{dq_1} = \frac{dq_2}{dq_1}$$

$\frac{dq_2}{dq_1}$ calculada sobre a FPP:

$$\frac{PMg_{q_{13}}}{PMg_{14}} = \frac{PMg_{23}}{PM_{2,4}}$$

$$\text{I} \quad \frac{PMg_{q_{23}}}{PMg_{q_{13}}} = \frac{PMg_{q_{24}}}{PMg_{q_{14}}} = -\lambda$$

$$w_3 = z_{1,3} + z_{2,3}$$

$$w_4 = z_{1,4} + z_{2,4}$$

$$q_1 = f_1(z_{1,3}, z_{1,4})$$

$$q_2 = f_2(z_{2,3}, z_{24})$$

$$\frac{dz_{13}}{dq_1} + \frac{dz_{23}}{dq_1} = 0 \quad \text{II}$$

$$\frac{dz_{14}}{dq_1} + \frac{dz_{24}}{dq_1} = 0 \quad \text{III}$$

$$PMg_{q_{13}} \frac{dz_{13}}{dq_1} + PMg_{q_{14}} \frac{dz_{14}}{dq_1} = 1 \quad \text{IV}$$

$$PMg_{q_{23}} \frac{dz_{23}}{dq_1} + PMg_{q_{24}} \frac{dz_{24}}{dq_1} = \frac{dq_2}{dq_1} \quad \text{V}$$

Substituindo II e III em V, obtemos

$$\frac{dq_2}{dq_1} = -PMg_{q_{2,3}} \frac{dz_{13}}{dq_1} - PMg_{q_{2,4}} \frac{dz_{14}}{dq_1} \quad \text{VI}$$

De I, obtemos: $PMg_{q_{23}} = -\lambda PMg_{q_{13}}$ e $PMg_{q_{24}} = -\lambda PMg_{q_{1,4}}$. Substituindo em VI vem:

$$\frac{dq_2}{dq_1} = \lambda PMg_{q_{13}} \frac{dz_{13}}{dq_1} + \lambda PMg_{q_{1,4}} \frac{dz_{14}}{dq_1} \Rightarrow \frac{dq_2}{dq_1} = \lambda \underbrace{\left[PMg_{q_{13}} \frac{dz_{13}}{dq_1} + PMg_{q_{14}} \frac{dz_{14}}{dq_1}\right]}_{=1 \ \text{IV}}$$

$$\frac{dq_2}{dq_1} = \lambda = -\underbrace{\frac{PMg_{q_{23}}}{PMg_{q_{13}}}}_{TMT_{1,2}} = -\underbrace{\frac{PMg_{q_{24}}}{PMg_{q_{14}}}}_{TMT_{1,2}}$$

$f_h(z_h)$

1.3 m consumidores e ℓ bens.

Escolher $\mathbf{x}_i$ para $i=1,\dots,m$ e ~~q_h~~ z_h para $h=1,\dots,\ell$ de modo a maximizar $U_1(\mathbf{x}_1)$ respeitando as condições

$U_i(\mathbf{x}_i) \geq \bar{u}_i$ para $i=2,\dots,m$

$\mathbf{x}_i \geq \mathbf{0}$ para $i=2,\dots,m$

$q_h \leq f_h(\mathbf{z}_h)$ para $h=1,\dots,\ell$

$\mathbf{z}_h \geq \mathbf{0}$ para $h=1,\dots,\ell$

$$\cancel{\sum_{i=1}^{m} \mathbf{x}_i - \sum_{j=1}^{n} \mathbf{y}_j - \mathbf{w} \leq 0}$$

$$\sum_{i=1}^{m} x_{ih} + \sum_{g=1}^{\ell} z_{g,h} - f_h(z_h) - w_h \leq 0$$

A função de Lagrange desse problema é

$$\mathcal{L} = U_1(\mathbf{x}_1) + \sum_{i=1}^{m} \lambda_i [U_i(\mathbf{x}_i) - \bar{u}_i] - \sum_{h=1}^{\ell} \mu_h \left[\sum_{i=1}^{m} x_{i,h} + \sum_{g=1}^{\ell} z_{g,h} - f_h(\mathbf{z}_h) - w_h \right] + \sum_{i=1}^{m} \sum_{h=1}^{\ell} \kappa_{i,h} x_{i,h} + \sum_{h=1}^{\ell} \sum_{g=1}^{\ell} \eta_{h,g} z_{h,g}$$

$\frac{\partial \mathcal{L}}{\partial x_{ih}} = 0 \Rightarrow \lambda_i UMg_{ih} - \mu_h + \kappa_{ih} = 0$ p/ $i=1,\dots,m$ $(\lambda_1 = 1)$ e $h=1,\dots,\ell$ e se $x_{i,h} > 0$, $\kappa_{i,h} = 0$

$\frac{\partial \mathcal{L}}{\partial z_{gh}} = 0 \Rightarrow -\mu_h + \mu_g PMg_{gh} + \eta_{h,g}$ p/ $h,g = 1,\dots,\ell$ e, se $z_{h,g} > 0$, $\eta_{h,g} = 0$

A função de Lagrange desse problema é

$$\mathcal{L} = U_1(\mathbf{x}_1) + \sum_{i=1}^{m} \lambda_i [U_i(\mathbf{x}_i) - \bar{u}_i] - \sum_{h=1}^{\ell} \mu_h \left[\sum_{i=1}^{m} x_{i,h} + \sum_{g=1}^{\ell} z_{g,h} - f_h(\mathbf{z}_h) - w_h \right] + \sum_{i=1}^{m} \sum_{h=1}^{\ell} \kappa_{i,h} x_{i,h} + \sum_{h=1}^{\ell} \sum_{g=1}^{\ell} \eta_{h,g} z_{h,g}$$

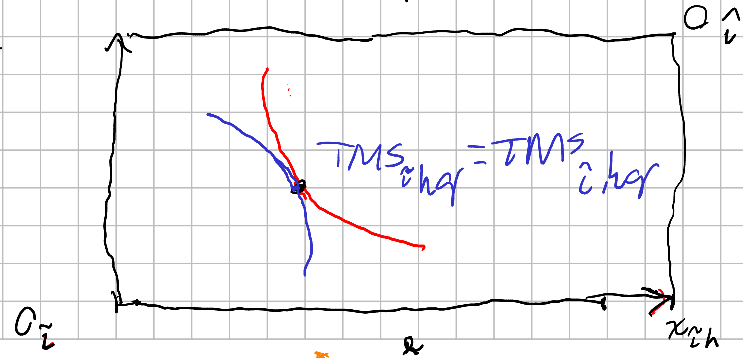
$$\frac{\partial \mathcal{L}}{\partial x_{ih}} = 0 \Rightarrow \lambda_i\, UMg_{ih} - \mu_h + \kappa_{ih} = 0 \quad p/\, i = 1, \dots, m \;\; (\lambda_1 = 1) \;\; e \;\; h = 1, \dots, \ell \;\; e \; se \; x_{i,h} > 0,\ \kappa_{i,h} = 0$$

$$\frac{\partial \mathcal{L}}{\partial z_{gh}} = 0 \Rightarrow -\mu_h + \mu_g\, PMg_{gh} + \eta_{h,g} \quad p/\, h, g = 1, \dots, \ell \;\; e, \; se \; z_{h,g} > 0,\ \eta_{h,g} = 0$$

Assumindo $x_{\tilde{i}h}$ e $x_{\tilde{i},g} > 0$ $(\kappa_{ih} = \kappa_{ig} = 0)$ e $x_{\hat{i}h}, x_{\hat{i}g} > 0$

$$\left.\begin{aligned} \lambda_{\tilde{i}}\, UMg_{\tilde{i}h} &= \mu_h = \lambda_{\hat{i}}\, UMg_{\hat{i}h} \\ \lambda_{\tilde{i}}\, UMg_{\tilde{i}g} &= \mu_g = \underline{\lambda_{\hat{i}}\, UMg_{\hat{i}g}} \end{aligned}\right\} \quad \frac{UMg_{\tilde{i}h}}{UMg_{\tilde{i}g}} = \frac{\mu_h}{\mu_g} = \frac{UMg_{\hat{i}h}}{UMg_{\hat{i}g}} \Rightarrow TMS_{\tilde{i}hg} = TMS_{\hat{i}hg}$$

$$w_g + f_g(\;) - \sum_{h=1}^{\ell} z_{hg} - \sum_{\substack{i \neq \hat{i} \\ i \neq \tilde{i}}}$$



$$\left| -w_h + f_h(\mathbf{z}_h) - \sum_{g=1}^{\ell} z_{gh} - \sum_{\substack{i \neq \hat{i} \\ i \neq \tilde{i}}} x_{ih} \right|$$

$$\mathcal{L} = U_1(\mathbf{x}_1) + \sum_{i=1}^{m} \lambda_i[U_i(\mathbf{x}_i) - \bar{u}_i] - \sum_{h=1}^{\ell} \mu_h \left[\sum_{i=1}^{m} x_{i,h} + \sum_{g=1}^{\ell} z_{g,h} - f_h(\mathbf{z}_h) - w_h\right] + \sum_{i=1}^{m}\sum_{h=1}^{\ell} \kappa_{i,h} x_{i,h} + \sum_{h=1}^{\ell}\sum_{g=1}^{\ell} \eta_{h,g} z_{h,g}$$

$\frac{\partial \mathcal{L}}{\partial z_{h,q}} = 0 \Rightarrow \mu_h PMg_{h,q} - \mu_q + \eta_{h,q} = 0$ p/ todo $h, q = 1, \dots, \ell$ e, se $z_{h,q} > 0$, $\eta_{h,q} = 0$

Se $\begin{cases} z_{h\tilde{q}},\ z_{h\hat{q}} > 0 \\ x_{i\tilde{q}},\ x_{i\hat{q}} > 0 \end{cases}$, então

$$\mu_h PMg_{h,\tilde{q}} = \mu_{\tilde{q}} = \lambda_i UMg_{i\tilde{q}}$$
$$\mu_h PMg_{h,\hat{q}} = \mu_{\hat{q}} = \lambda_i UMg_{i\hat{q}}$$

$$\frac{PMg_{h,\tilde{q}}}{PMg_{h\hat{q}}} = \frac{UMg_{i\tilde{q}}}{UMg_{i\hat{q}}}$$

$$TMST_{h\tilde{q}\hat{q}} = TMS_{i\tilde{q}\hat{q}}$$

Se, além disso, $z_{\tilde{h},\hat{q}}$, $z_{\tilde{h}\tilde{q}}$, então

$$\frac{PMg_{h\tilde{q}}}{PMg_{h\hat{q}}} = \frac{UMg_{i\tilde{q}}}{UMg_{i\hat{q}}} = \frac{PMg_{\tilde{h}\tilde{q}}}{PMg_{\tilde{h}\hat{q}}} \Rightarrow \frac{PMg_{h\hat{q}}}{PMg_{\tilde{h}\hat{q}}} = \frac{PMg_{h\tilde{q}}}{PMg_{\tilde{h}\tilde{q}}} = TMT_{\tilde{h}\hat{h}}$$

$$\mu_h PMg_{h,\tilde{q}} = \mu_{\tilde{q}} = \mu_{\tilde{h}} PMg_{\tilde{h}\tilde{q}} \Rightarrow \frac{\mu_h}{\mu_{\tilde{h}}} = \frac{PMg_{\tilde{h}\tilde{q}}}{PMg_{h\tilde{q}}} \Rightarrow \frac{UMg_{ih}}{UMg_{i\tilde{h}}} = \frac{PMg_{\tilde{h}\tilde{q}}}{PMg_{h\tilde{q}}}$$

$$TMS_{ih\tilde{h}} = TMT_{h\tilde{h}}$$

1.3 m consumidores, n produtores e ℓ bens

Escolher $\mathbf{x}_i$ para $i = 1, \ldots, m$ e $\mathbf{y}_j$ para $j = 1, \ldots, n$ de modo a maximizar $U_1(\mathbf{x}_1)$ respeitando as condições

$$U_i(\mathbf{x}_i) \geq \bar{u}_i \quad \text{para } i = 2, \ldots, m$$

$$\mathbf{x}_i \geq \mathbf{0}$$

$$F_j(\mathbf{y}_j) \leq 0 \quad \text{para } j = 1, \ldots, n$$

$$\sum_{i=1}^{m} \mathbf{x}_i - \sum_{j=1}^{n} \mathbf{y}_j - \mathbf{w} \leq 0$$

$F_j(y_j)$ = função de transformação do produtor j.

$y_j = (y_{j1}, \ldots, y_{j\ell})$

$$\sum_{i=1}^{n} x_{ih} \leq \sum_{j=1}^{m} y_{jh} + w_h \quad p/\ h = 1, \ldots, \ell$$

A função de Lagrange desse problema é

$$\mathcal{L} = U_1(\mathbf{x}_1) + \sum_{i=1}^{m} \lambda_i [U_i(\mathbf{x}_i) - \bar{u}_i] - \sum_{h=1}^{\ell} \mu_h \left[\sum_{i=1}^{m} x_{i,h} - \sum_{j=1}^{n} y_{j,h} - w_h \right] + \sum_{i=1}^{m} \sum_{h=1}^{\ell} \kappa_{i,h} x_{i,h} - \sum_{j=1}^{m} \eta_j F_j(\mathbf{y}_j)$$

Cond: 1ª ordem

$$\frac{\partial \mathcal{L}}{\partial x_{ih}} = 0 \Rightarrow \lambda_i \, UMg_{ih} - \mu_h + k_{ih} = 0 \quad p/\ i = 1, \ldots, m \text{ e } h = 1, \ldots, \ell \text{ com } \lambda_1 = 1 \text{ e, se } x^*_{ih} > 0,\ k_{ih} = 0$$

$$\frac{\partial \mathcal{L}}{\partial y_{jh}} = 0 \Rightarrow \mu_h - \eta_j \frac{\partial F_j}{\partial y_{jh}} = 0 \quad p/\ h = 1, \ldots, \ell \text{ e } j = 1, \ldots, m$$

$$\mathcal{L} = U_1(\mathbf{x}_1) + \sum_{i=1}^{m} \lambda_i [U_i(\mathbf{x}_i) - \bar{u}_i] - \sum_{h=1}^{\ell} \mu_h \left[\sum_{i=1}^{m} x_{i,h} - \sum_{j=1}^{n} y_{j,h} - w_h \right] + \sum_{i=1}^{m}\sum_{h=1}^{\ell} \kappa_{i,h} x_{i,h} - \sum_{j=1}^{m} \eta_j F_j(\mathbf{y}_j)$$

$F_j(\ ) = 0$

Cond. 1as ordem

(I) $\frac{\partial \mathcal{L}}{\partial x_{ih}} = 0 \Rightarrow \lambda_i UMg_{ih} - \mu_h + k_{ih} = 0$ p/ $i = 1, \ldots, m$ e $h = 1, \ldots, \ell$ com $\lambda_1 = 1$ e, se $x^*_{ih} > 0$, $k_{ih} = 0$

(II) $\frac{\partial \mathcal{L}}{\partial y_{jh}} = 0 \Rightarrow \mu_h - \eta_j \frac{\partial F_j}{\partial y_{jh}} = 0$ p/ $h = 1, \ldots, \ell$ e $j = 1, \ldots, m$

Assuma que $x_{i',h'}$, $x_{i''h'}$, $x_{i'h''}$ e $x_{i''h''} > 0$, então, usando (I),

(a) $\lambda_{i'} UMg_{i'h'} = \mu_{h'}$

(b) $\lambda_{i''} UMg_{i''h'} = \mu_{h'}$

(c) $\lambda_{i'} UMg_{i'h''} = \mu_{h''}$

(d) $\lambda_{i''} UMg_{i''h''} = \mu_{h''}$

a/c: $\frac{UMg_{i'h'}}{UMg_{i'h''}} = \frac{\mu_{h'}}{\mu_{h''}} = \frac{UMg_{i''h'}}{UMg_{i''h''}}$

$TMS_{i'h'h''} = TMS_{i''h'h''}$

da cond. (II), p/ as empresas j' e j'' deve valer

$$\begin{cases} \eta_{j'} \frac{\partial F_{j'}}{\partial y_{h'}} = \mu_{h'} \\ \eta_{j'} \frac{\partial F_{j'}}{\partial y_{h''}} = \mu_{h''} \end{cases} \qquad \begin{cases} \eta_{j''} \frac{\partial F_{j''}}{\partial y_{h'}} = \mu_{h'} \\ \eta_{j''} \frac{\partial F_{j''}}{\partial y_{h''}} = \mu_{h''} \end{cases}$$

$\frac{\partial F_{j'}/\partial y_{h'}}{\partial F_{j'}/\partial y_{h''}} = \frac{\partial F_{j''}/\partial y_{h'}}{\partial F_{j''}/\partial y_{h''}} = \frac{\mu_{h'}}{\mu_{h''}} = TMS_{i'h'h''} = TMS_{i''h'h''}$

$TMT_{j'h'h''} = TMT_{j''h'h''} = TMS_{ih'h''}$

2: Equilíbrio geral

Em uma economia com:

l bens,

n consumidores c/ função de util. $U_i(x_i)$ ← $x_i = (x_{i1}, x_{i2}, \dots, x_{il})$

e dotações iniciais $\omega_1, \omega_2, \dots, \omega_m$ $[\omega_i = (\omega_{i1}, \omega_{i2}, \dots, \omega_{il})]$

e

m empresas c/ funções de transf $F_j(y_j)$, $(y_{j1}, y_{j2}, \dots, y_{jm})$

um vetor de preços p^* e uma alocação de consumo $x^*_1, x^*_2, \dots, x^*_m$ e de produção $y^*_1, y^*_2, \dots, y^*_m$ configuram um equilíbrio geral caso:

- $\sum_{i=1}^{m} x_i^* \leq \sum_{j=1}^{n} y_j + \sum_{i=1} \omega_i$ ← $p_1^* x_{i1} + p_2^* x_{i2} + \dots \leq m$ (riqueza) (renda)
- x_i^* maximiza $U_i(x_i)$ dada a restrição $p^* \cdot x_i \leq p^* \cdot \omega_i + \sum_{j=1}^{m} s_{ij}\, p \cdot y_j^*$ ← $p_1 \omega_{i1} + p_2 \omega_{i2} + \dots + p_l \omega_{il}$

  e
- y_j^* maximiza $\overbrace{p^* \cdot y_j}^{\text{Lucro}}$ dada a restrição $F(y_j) \leq 0$ ←

$p_1^* y_{j1} + p_2^* y_{j2} + \dots$

$TMT_{jhh'} = \frac{p_h}{p_{h'}}$

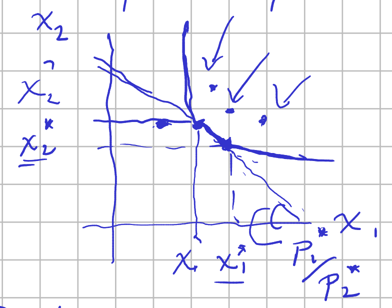
$TMS_{ihh'} = \frac{p_h}{p_{h'}}$

$p \cdot y_j = p_1 y_{j1} + p_2 y_{j2} + \dots + p_l y_{jl}$ = Lucro da emp.

1º Teorema do bem estar social:

· Se as preferências dos consumidores são localmente não saciáveis, então toda alocação de equilíbrio geral competitivo é também uma alocação eficiente.

· Sejam $p^*, x_1^*, x_2^*, \dots, x_m^*, y_1^*, y_2^*, \dots, y_n^*$, um vetor de preços e alocações de consumo e de produção em um equilíbrio geral competitivo.



Se as preferências são localmente não saciáveis, então, p/ $i = 1, \dots, m$

$$\Rightarrow p^* x_i^* = p^*\cdot\omega_i + \sum_{j=1}^{n} s_{ij}\, p^*\cdot y_j^* \quad \text{(II)}$$

· Suponha que tal alocação de equilíbrio não seja eficiente. Então deverá haver uma alocação de consumo e produção $x_1', x_2', \dots, x_m', y_1', y_2', \dots, y_n'$ tal que $y_j' \in Y_j$, $j = 1, \dots, n$

$$\sum_{i=1}^{m} x_i' \le \sum_{i=1}^{m} \omega_i + \sum_{j=1}^{n} y_j' \quad (*)$$

e, p/ todo $i = 1, \dots, m$, $x_i' \succsim x_i^*$ e, para ao menos um $i \in \{1, \dots, m\}$, $x_i' \succ x_i^*$

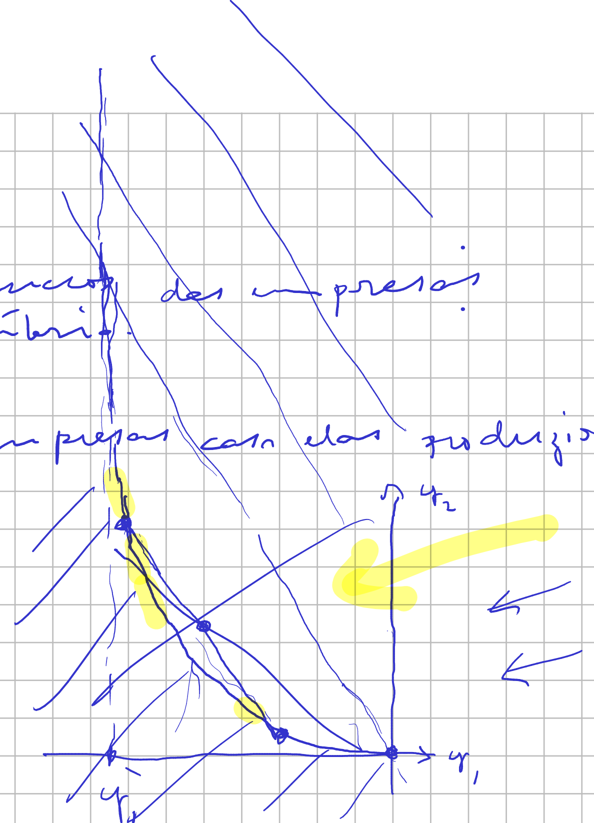
isso implica que p/ $i = 1, \dots, m$, $p^*\cdot x_i' \ge p^*\cdot x_i^*$ e p/ algum $i \in \{1, \dots, m\}$ $p^*\cdot x_i' > p^*\cdot x_i^* = p^*\cdot\omega_i + \sum_{j=1}^{n} s_{ij}\, p^*\cdot y_j^*$

$\sum_{i=1}^{m} p^*\cdot x_i' > \sum_{i=1}^{m} p^*\cdot x_i^*$. Usando (I) e (II), $p^*\cdot\sum_{i=1}^{m}\omega_i + p^*\cdot\sum_{j=1}^{n} y_j' > \sum_{i=1}^{m}\left[p^*\cdot\omega_i + \sum_{j=1}^{n} s_{ij}\, p^*\cdot y_j^*\right] \Rightarrow \cancel{\sum_{i=1}^{m} p^*\cdot\omega_i} + \sum_{j=1}^{n} p^*\cdot y_j' > \cancel{\sum_{i=1}^{m} p^*\cdot\omega_i} + \sum_{i=1}^{m}\sum_{j=1}^{n} s_{ij}\, p^*\cdot y_j^*$

$$\sum_{j=1}^{n} p^*\cdot y'_j > \sum_{j=1}^{n}\sum_{i=1}^{m} s_{ij}\, p^*\cdot y_j^* \Rightarrow \sum_{j=1}^{n} p^*\cdot y'_j > \sum_{j=1}^{n} p^*\cdot y_j^* \underbrace{\sum_{i=1}^{m} s_{ij}}_{=1} \Rightarrow \sum_{j=1}^{n} p^*\cdot y'_j > \sum_{j=1}^{n} p^*\cdot y_j^*$$

$\sum_{j=1}^{n} p^*\cdot y_j^*$: soma dos lucros das empresas em equilíbrio.

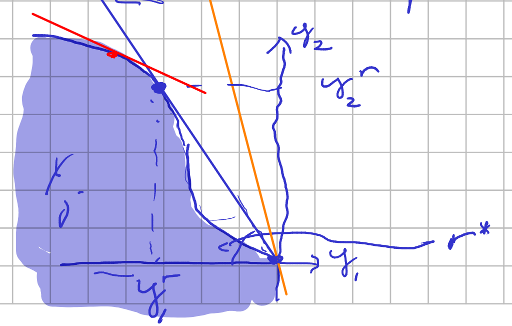
$\sum_{j=1}^{n} p^*\cdot y'_j$: soma dos lucros das empresas caso elas produzissem $y'_1, \dots, y'_m$



∴ Para, ao menos uma empresa, $p^*\cdot y'_j > p^*\cdot y_j^*$, o que só é factível caso $y'_j \notin Y_j$, o viola a hipótese de que y'_j é factível, ou caso y_j^* não for um vetor de prod. líq. que maximize o lucro da empresa j, o que viola nossa hipótese inicial.

Existência do equilíbrio:

É cond. suficiente para garantir a existência do EGC que as preferências de cada um dos consumidores sejam convexas e que os conjuntos de produção de cada firma sejam convexos.



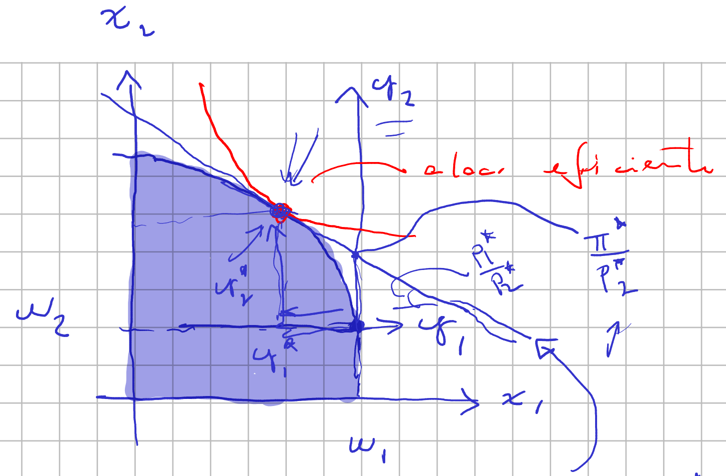
se $\frac{p_1}{p_2} > r^*$, $y_j^*(p_1, p_2) = (0,0)$

se $\frac{p_1}{p_2} = r^*$, $y_j^*(p_1, p_2) = \{(0,0), (y_1^r, y_2^r)\}$

se $\frac{p_1}{p_2} < r^*$ $y_{j1}^*(p_1, p_2) < y_1^r$ e $y_{j2}^* > y_2^r$

2º Teorema do bem estar social:

Se as preferências de todos os consumidores forem convexas e os conjuntos de produção de todos os produtores também forem convexos, toda alocação eficiente será também uma alocação de equilíbrio para uma adequada redistribuição das dotações iniciais entre os consumidores.



$$p_1x_1 + p_2x_2 \leq p_1\omega_1 + p_2\omega_2 + \pi^*$$